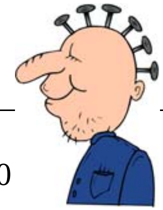
Algebra 2 – Things to Remember!



Exponents:

$x^0 = 1$ $x^{-m} = \dfrac{1}{x^m}$

$x^m \bullet x^n = x^{m+n}$ $(x^n)^m = x^{n\bullet m}$

$\dfrac{x^m}{x^n} = x^{m-n}$ $\left(\dfrac{x}{y}\right)^n = \dfrac{x^n}{y^n}$

$(xy)^n = x^n \bullet y^n$

Complex Numbers:

$\sqrt{-1} = i$ $\sqrt{-a} = i\sqrt{a};\, a \ge 0$

$i^2 = -1$ $i^{14} = i^2 = -1$ divide exponent by 4, use remainder, solve.

$(a+bi)$ conjugate $(a-bi)$

$(a+bi)(a-bi) = a^2 + b^2$

$|a+bi| = \sqrt{a^2+b^2}$ absolute value=magnitude

Logarithms

$y = \log_b x \Leftrightarrow x = b^y$

$\ln x = \log_e x$ natural log

$e = 2.71828\ldots$

$\log x = \log_{10} x$ common log

Change of base formula:

$\log_b a = \dfrac{\log a}{\log b}$

Properties of Logs:

$\log_b b = 1$ $\log_b 1 = 0$

$\log_b (m \bullet n) = \log_b m + \log_b n$

$\log_b \left(\dfrac{m}{n}\right) = \log_b m - \log_b n$

$\log_b (m^r) = r \log_b m$

Domain: $\log_b x$ is $x > 0$

Factoring:

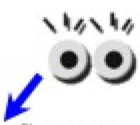
Look to see if there is a GCF (greatest common factor) first. $ab + ac = a(b+c)$

$x^2 - a^2 = (x-a)(x+a)$

$(x+a)^2 = x^2 + 2ax + a^2$

$(x-a)^2 = x^2 - 2ax + a^2$

Factor by Grouping:



$x^3 + 2x^2 - 3x - 6$

$(x^3 + 2x^2) - (3x + 6)$ group

$x^2(x+2) - 3(x+2)$ factor each

$(x^2 - 3)(x+2)$ factor

Exponentials $e^x = \exp(x)$

$b^x = b^y \rightarrow x = y$ ($b > 0$ and $b \ne 1$)

If the bases are the same, set the exponents equal and solve.

Solving exponential equations:

1. Isolate exponential expression.
2. Take *log* or *ln* of both sides.
3. Solve for the variable.

$\ln(x)$ and e^x are inverse functions

$\ln e^x = x$ $e^{\ln x} = x$

$\ln e = 1$ $e^{\ln 4} = 4$

$e^{2\ln 3} = e^{\ln 3^2} = 9$

Quadratic Equations: $ax^2 + bx + c = 0$ (Set = 0.)

Solve by factoring, completing the square, quadratic formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac > 0$ two real unequal roots

$b^2 - 4ac = 0$ repeated real roots

$b^2 - 4ac < 0$ two complex roots

Square root property: If $x^2 = m$, then $x = \pm\sqrt{m}$

Completing the square: $x^2 - 2x - 5 = 0$

1. If other than one, divide by coefficient of x^2
2. Move constant term to other side $x^2 - 2x = 5$
3. Take half of coefficient of x, square it, add to both sides $x^2 - 2x + \boxed{1} = 5 + \boxed{1}$
4. Factor perfect square on left side. $(x-1)^2 = 6$
5. Use square root property to solve and get two answers. $x = 1 \pm \sqrt{6}$

Sum of roots: $r_1 + r_2 = -\dfrac{b}{a}$ **Product of roots**: $r_1 \bullet r_2 = \dfrac{c}{a}$

Variation: always involves the constant of proportionality, k. Find k, and then proceed.

Direct variation: $y = kx$

Inverse variation: $y = \dfrac{k}{x}$

Varies jointly: $y = kxj$

Combo: Sales vary directly with advertising and inversely with candy cost. $y = \dfrac{ka}{c}$

Absolute Value: $|a| > 0$

$|a| = \begin{cases} a; & a \ge 0 \\ -a; & a < 0 \end{cases}$

$|m| = b \Rightarrow m = -b$ or $m = b$

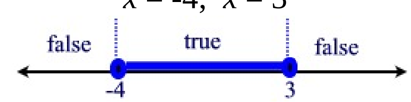
$|m| < b \Rightarrow -b < m < b$

$|m| > b \Rightarrow m > b$ or $m < -b$

Inequalities: $x^2 + x - 12 \le 0$ Change to =, factor, locate critical points on number line, check each section.

$(x + 4)(x - 3) = 0$

$x = -4;\ x = 3$



ANSWER: $-4 \le x \le 3$ or $[-4, 3]$ (in interval notation)

Radicals: Remember to use fractional exponents.

$$\sqrt[a]{x} = x^{\frac{1}{a}} \qquad x^{\frac{m}{n}} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m$$

$$\sqrt[n]{a^n} = a \qquad \sqrt[n]{ab} = \sqrt[n]{a}\bullet\sqrt[n]{b} \qquad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

Simplify: look for perfect powers.

$$\sqrt{x^{12}y^{17}} = \sqrt{x^{12}y^{16}y} = x^6y^8\sqrt{y}$$

$$\sqrt[3]{72x^9y^8z^3} = \sqrt[3]{8\bullet 9x^9y^6y^2z^3} = 2x^3y^2z\sqrt[3]{9y^2}$$

Use conjugates to rationalize denominators:

$$\frac{5}{2+\sqrt{3}}\bullet\frac{2-\sqrt{3}}{2-\sqrt{3}} = \frac{10-5\sqrt{3}}{4-2\sqrt{3}+2\sqrt{3}-\sqrt{9}} = 10-5\sqrt{3}$$

Equations: isolate the radical; square both sides to eliminate radical; combine; solve.

$$2x-5\sqrt{x}-3=0 \quad\rightarrow\quad (2x-3)^2 = (5\sqrt{x})^2$$

$$4x^2-12x+9 = 25x \quad\rightarrow\quad solve: x=9;\ x=1/4$$

CHECK ANSWERS. Answer only *x* = 9.

Functions: A function is a set of ordered pairs in which each *x*-element has only ONE *y*-element associated with it.

Vertical Line Test: is this graph a function?
Domain: *x*-values used; **Range:** *y*-values used
Onto: all elements in B used.
1-to-1: no element in B used more than once.
Composition: $(f\circ g)(x) = f(g(x))$
Inverse functions *f* & *g*: $f(g(x)) = g(f(x)) = x$
Horizontal line test: will inverse be a function?

Transformations:
$-f(x)$ over *x*-axis; $f(-x)$ over y-axis
$f(x+a)$ horizontal shift; $f(x)+a$ vertical shift
$f(ax)$ stretch horizontal; $af(x)$ stretch vertical

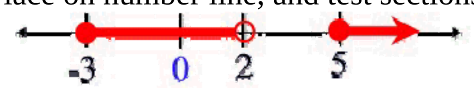
Working with Rationals (Fractions):
Simplify:
remember to look for a factoring of -1:

$$\frac{3x-1}{1-3x} = \frac{-1(\cancel{-3x+1})}{\cancel{1-3x}} = -1$$

Add: Get the common denominator.
Factor first if possible:
Multiply and Divide: Factor First

Rational Inequalities

$$\frac{x^2-3x-15}{x-2} \geq 0$$ The critical values from factoring the numerator are -3, 5.
The denominator is zero at *x* = 2.
Place on number line, and test sections.



Sequences
Arithmetic: $a_n = a_1 + (n-1)d$

$$S_n = \frac{n(a_1+a_n)}{2}$$

Geometric: $a_n = a_1\bullet r^{n-1}$

$$S_n = \frac{a_1(1-r^n)}{1-r}$$

Recursive: Example:
$a_1 = 4;\quad a_n = 2a_{n-1}$

Binomial Theorem:

$$(a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^{n-k}b^k$$

Solving Rational Equations:
Get rid of the denominators by mult. all terms by common denominator.

$$\frac{22}{2x^2-9x-5} - \frac{3}{2x+1} = \frac{2}{x-5}$$

multiply all by $2x^2-9x-5$ *and get*

$$22-3(x-5) = 2(2x+1)$$
$$22-3x+15 = 4x+2$$
$$37-3x = 4x+2$$
$$35 = 7x$$
$$5 = x$$

Great! But the only problem is that
x = 5 does not CHECK!!!! There is no solution.
Extraneous root.
Motto: Always CHECK ANSWERS.

Equations of Circles: $x^2+y^2=r^2$ center origin
$(x-h)^2+(y-k)^2=r^2$ center at (*h*,*k*)
$x^2+y^2+Cx+Dy+E=0$ general form

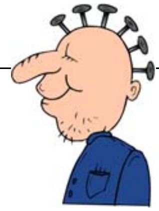
Complex Fractions:
Remember that the fraction bar means divide:
Method 1: Get common denominator top and bottom

$$\frac{\frac{2}{x^2}-\frac{4}{x}}{\frac{4}{x}-\frac{2}{x^2}} = \frac{\frac{2-4x}{x^2}}{\frac{4x-2}{x^2}} = \frac{2-4x}{x^2} \div \frac{4x-2}{x^2} = \frac{\cancelto{-1}{2-4x}}{\cancel{x^2}}\bullet\frac{\cancel{x^2}}{\cancel{4x-2}} = -1$$

Method 2: Mult. all terms by common denominator for all.

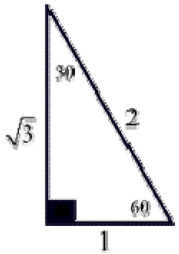
$$\frac{\frac{2}{x^2}-\frac{4}{x}}{\frac{4}{x}-\frac{2}{x^2}} = \frac{x^2\bullet\frac{2}{x^2}-x^2\bullet\frac{4}{x}}{x^2\bullet\frac{4}{x}-x^2\bullet\frac{2}{x^2}} = \frac{2-4x}{4x-2} = -1$$

Trigonometry – Things to Remember!

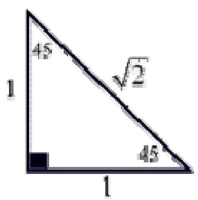


Arc Length of a Circle = θr (in radians)

Special Right Triangles



30º-60º-90º triangle
side opposite 30º = ½ hypotenuse
side opposite 60º = ½ hypotenuse $\sqrt{3}$



45º-45º-90º triangle
hypotenuse = leg $\sqrt{2}$
leg = ½ hypotenuse $\sqrt{2}$

Law of Sines: uses 2 sides and 2 angles

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Has an ambiguous case.

Law of Cosines: uses 3 sides and 1 angle

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Area of triangle: $A = \frac{1}{2}ab\sin C$
Area of parallelogram: $A = ab\sin C$

Pythagorean Identities:

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

Radians and Degrees

Change to radians multiply by $\dfrac{\pi}{180}$

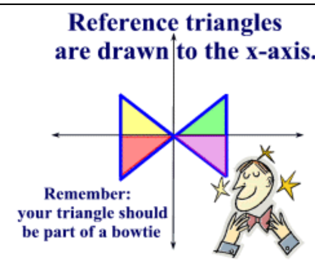
Change to degrees multiply by $\dfrac{180}{\pi}$

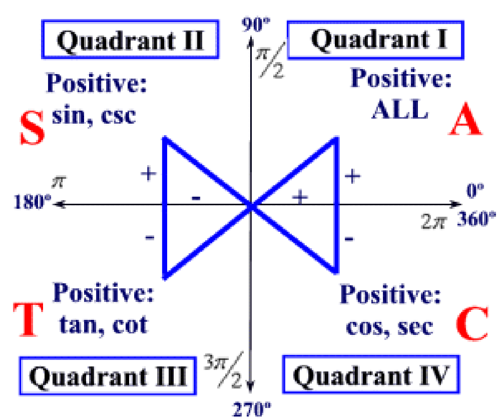
Quadrantal angles – 0, 90, 180, 270

CoFunctions: examples

$\sin\theta = \cos(90^\circ - \theta)$; $\tan\theta = \cot(90^\circ - \theta)$

Inverse notation:
arcsin(x) = $\sin^{-1}(x)$
arccos(x) = $\cos^{-1}(x)$
arctan(x) = $\tan^{-1}(x)$





Trig Functions

$$\sin\theta = \frac{o}{h};\quad \cos\theta = \frac{a}{h};\quad \tan\theta = \frac{o}{a}$$

$$\csc\theta = \frac{h}{o};\quad \sec\theta = \frac{h}{a};\quad \cot\theta = \frac{a}{o}$$

Reciprocal Functions

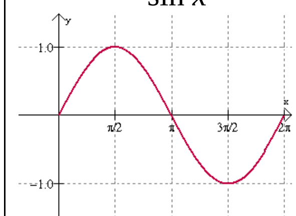
$$\sin\theta = \frac{1}{\csc\theta};\quad \cos\theta = \frac{1}{\sec\theta};\quad \tan\theta = \frac{1}{\cot\theta}$$

$$\csc\theta = \frac{1}{\sin\theta};\quad \sec\theta = \frac{1}{\cos\theta};\quad \cot\theta = \frac{1}{\tan\theta}$$

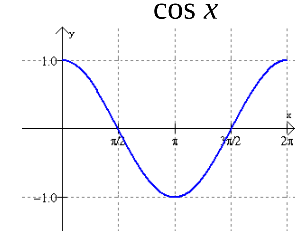
$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Trig Graphs

sin x



cos x



sinusoidal curve = any curve expressed as
$y = A\sin(B(x - C)) + D$

amplitude (A) = ½ | max – min| *(think height)*

period = horizontal length of 1 complete cycle

frequency (B) = number of cycles in 2π (period)

horizontal shift (C) – movement left/right

vertical shift (D) – movement up/down

Statistics and Probability – Things to Remember!

Statistics:

$$mean = \bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

median = middle number in ordered data
mode = value occurring most often

range = difference between largest and smallest

mean absolute deviation (MAD):

$$population\ MAD = \frac{1}{n}\sum_{i=1}^{n}\left|x_i - \bar{x}\right|$$

variance:

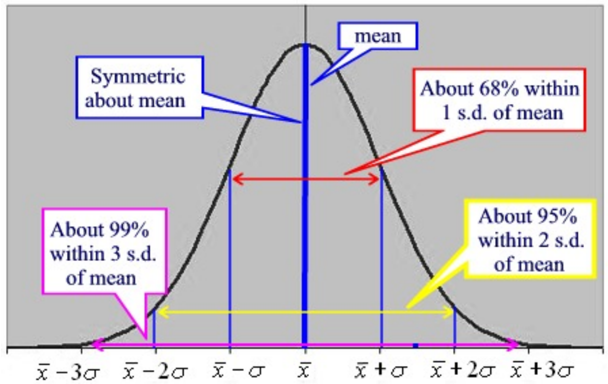
$$population\ \text{variance} = (\sigma x)^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2$$

standard deviation:
population standard deviation =

$$\sigma x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}$$

Sx = sample standard deviation
σ_x = population standard deviation

Normal Distribution and Standard Deviation



Binomial Probability

${}_nC_r \bullet p^r \bullet q^{n-r}$ **"exactly"** *r* times

or $\binom{n}{r} \bullet p^r \bullet (1-p)^{n-r}$

[TI Calculator: binompdf(*n*, *p*, *r*)]

When computing **"at least"** and **"at most"** probabilities, it is necessary to consider, in addition to the given probability,

• all probabilities larger than the given probability (**"at least"**)
[TI Calculator: 1 – binomcdf(*n*, *p*, *r*-1)]

• all probabilities smaller than the given probability (**"at most"**)
[TI Calculator: binomcdf(*n*, *p*, *r*)]

Probability

Permutation: without replacement and order matters

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combination: without replacement and order does not matter

$${}_nC_r = \binom{n}{r} = \frac{{}_nP_r}{r!} = \frac{n!}{r!(n-r)!}$$

Empirical Probability

$$P(E) = \frac{\#\text{ of times event } E \text{ occurs}}{\text{total \# of observed occurrences}}$$

Theoretical Probability

$$P(E) = \frac{n(E)}{n(S)} = \frac{\#\text{ of outcomes in } E}{\text{total \# of outcomes in } S}$$

P(A and B) = P(A)•P(B)
for independent events
P(A and B) = P(A)•P(B| A)
for dependent events

P(A') = 1 – P(A)

P(A or B) = P(A) + P(B) – P(A and B)
for not mutually exclusive

P(A or B) = P(A) + P(B)
for mutually exclusive

$$P(B\,|\,A) = \frac{P(A \text{ and } B)}{P(A)} \quad \text{(conditional)}$$